Logistic regression: binomial (+multinomial, ordered, conditional)

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RESMA Course Data Analysis & Report #6

OLS assumptions

- Dependent variable is a score:
  - Continuous
  - Unbounded
  - Varies linear with the predictor variable.
- However, our variables in fact are often a choice among categories:
  - Discrete
  - Limited range
  - Often have a (partial) rank-order and sometimes have a distance
  - May associate with predictor variables in irregular (non-linear) ways.
- Important special case: outcome variable is 0/1 (binary, binomial). Not: dummy.

Variations in SPSS

- LOGISTIC: for binary outcome variables
- NOMREG: for multinomial outcome variables.
- PLUM: for ordered multinomial outcome variables.
- LOGLINEAR: all sort of models, but discrete independent variables.
- All these programs will do binary logistic regression as a special case. The coefficients may look different (see O'Connell and excercise).

0/1 dichotomy as dependent variable in OLS

- If you use OLS to model 0/1 outcomes (the 'linear probability' model), the following problems will arise:
  - The OLS assumption of homoskedasticity will not apply: the variation is very small at the extremes. This will bias the coefficients and invalidate the SE estimates.
  - Predicted values may occur outside the 0/1 range.
- Both problems are most severe when you are modeling a variable that has an expected value (=mean) close to 0 or 1. This happens often in event analysis (next course).
- However, when you are modeling a variable in the 0.20..0.80 range, the 'linear probability model' is in fact quite useful, at least to look at.

Example: being a student in ISSP06

- Voorbeeld: studerend in ISSP2006. Data voor leeftijd 18-64, N=1575. Gemiddelde is 3.2%, oftewel .032.
- Student zijn is zeer sterk gedifferentieerd naar leeftijd. Het komt eigenlijk alleen bij jonge mensen voor.
- OLS Model: STUDENT = 0.239 - .0047 * AGECAT.
- De slope is zeer significant: t = 12.8.
- Data worden zeer slecht gerepresenteerd door het lineaire probabiliteitsmodel; verwachte kans op student zijn voor ouderen wordt negatief (-4%).

Logit = ln(odds)

- Dependent variable is a probability P (if Y=1) and 1-P (if Y=0).
- Odds [kansverhouding]: P / (1-P).
- P ≠ odds! However, odds is close to P at very low P.
- Logit [log kansverhouding]: ln(P/(1-P)).
- In any kind of logistic regression of nominal data, the dependent variable is the logit.
- So you should familiarize yourself with logarithms.

Logarithms (1)

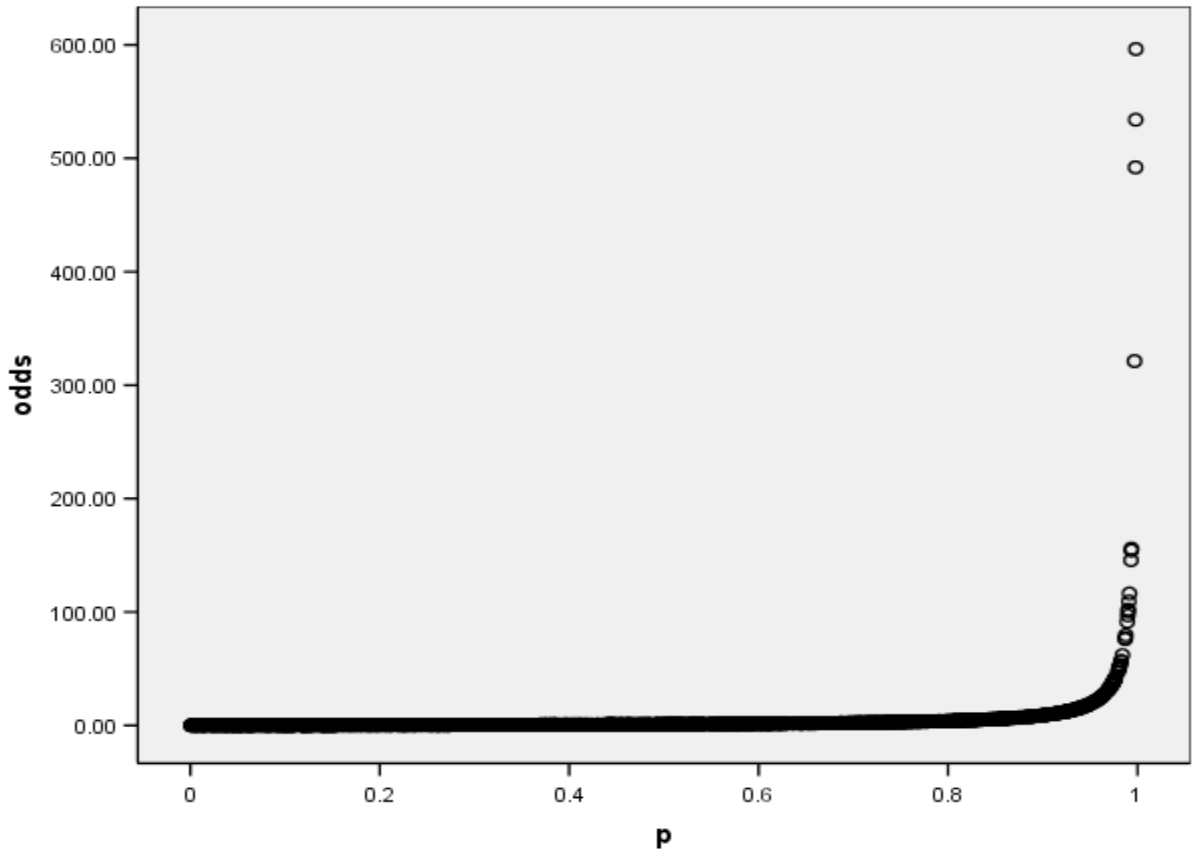
- Logaritme X: tot welke **macht** moet je een **grondtal** verheffen om X te verkrijgen. Zie bv.: http://nl.wikipedia.org/wiki/Logaritme.
- Grondtal 10: $^{10}\log(100)=2$
- Grondtal 2: $^{2}\log(64) = 6$.
- Grondtal e = exp = 2.718: $^{e}\log(100) = \ln(100) = 4.61$.
- Ln(a*b) = ln(a)+ln(b)
- exp(a+b) = exp(a)*exp(b)
- Ln(exp(a+b)) = a+b
- Vermenigvuldigen → optellen
- Delen → Aftrekken
- Machtverheffen → vermenigvuldigen of delen

Logarithms (2)

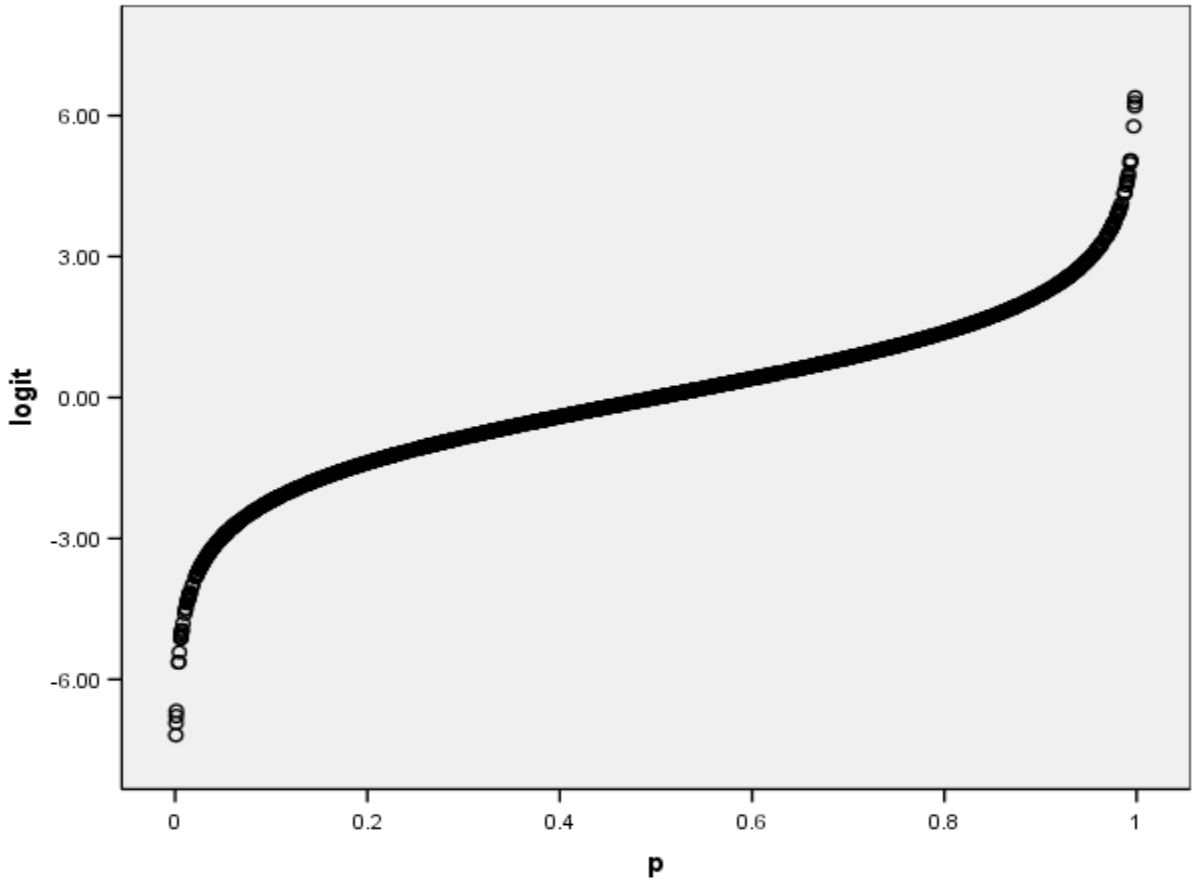
- Ln(2.718) = 1
- Ln(2) = .69
- Ln(1) = 0
- Ln(.5) = -.69
- Ln (0) = oneindig = onbepaald

- Exp(1) = 2.718
- Exp(0) = 1

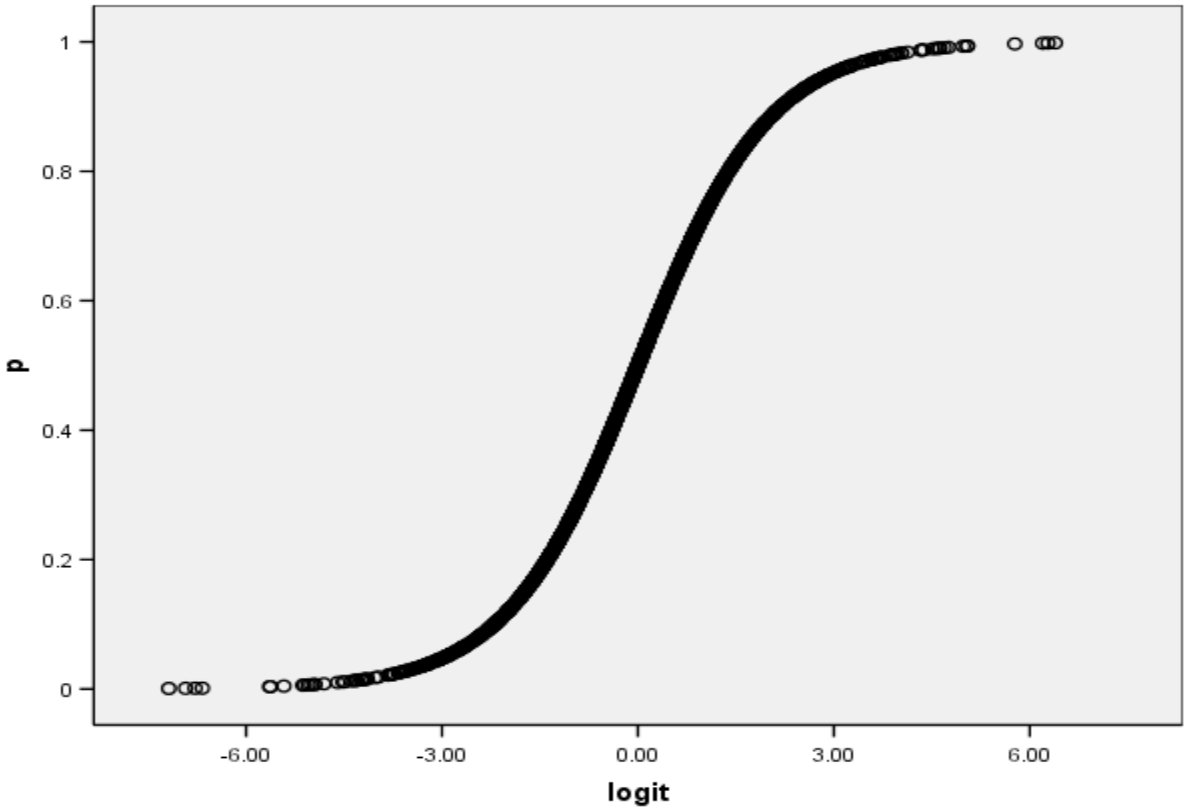
P versus odds



P versus logit (LN transformation)



Logit versus P



Logit, odds, and P

- Logit = ln(odd) = ln (P/(1-P)).
- Exp(logit) → odds.
- Ln(odds) → logit.
- P = 1 / (1+exp(-logit))
- (this transformation will SPSS do for you in ‘predicted value’).

Logistic regression in SPSS

- **`Analyze > regression > binary logistic`**
- The syntax of the model is different from REGR, but similar to UNIANOVA:
  - **`Logistic Y with X1 X2`**. [additive, linear]
  - **`Logistic Y with X1 X2 C1 /cat=C1`**. [+ categorical]
  - **`Logistic Y with X1 X2 C1 X2*C1 /cat=C1`**. [+interaction]
- So, syntax provides for (A) automatic creation of dummy variables, and (B) automatic creation of interaction terms.

Tabel ‘Case Processing Summary’

Case Processing Summary

Unweighted Cases [a]		N	Percent
Selected Cases	Included in Analysis	1575	100.0
	Missing Cases	0	.0
	Total	1575	100.0
Unselected Cases		0	.0
Total		1575	100.0

a. If weight is in effect, see classification table for the total number of cases.

Tabel ‘Dependent Variable Encoding’

Dependent Variable Encoding

Original Value	Internal Value
0	0
1	1

Tabel ‘Omnibus Tests of Model coefficients’

Omnibus Tests of Model Coefficients

		Chi-square	df	Sig.
Step 1	Step	187.933	1	.000
	Block	187.933	1	.000
	Model	187.933	1	.000

Tabel ‘Model Summary’

Model Summary

Step	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
1	255.462[a]	.112	.458

a. Estimation terminated at iteration number 9 because parameter estimates changed by less than .001.

Tabel ‘Classification Table’

Classification Table[a]

Observed		Predicted		
		unempl		Percentage Correct
		0	1	
Step 1 unempl	0	1518	7	99.5
	1	35	15	30.0
Overall Percentage				97.3

a. The cut value is .500

Some good advice

- Do not leave the coding of the dependent variable to the program.
- Missing values always need scrutiny. There is no pairwise option. Use substitution to see effects of missing values patterns.
- Like in OLS, life becomes happier when you code your independent variables using a 0 and an interpretable unit.
- Significance of individual coefficients: t = b/SE.
- With some practice, the multiplicative coefficients are easier to talk about than the logistic ones.

Table 'Variables in the Equation':

Variables in the Equation

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1[a]	agecat	-.269	.029	84.845	1	.000	.764
	Constar	5.310	.783	46.034	1	.000	202.396

a. Variable(s) entered on step 1: agecat.

Logistische en multiplicatieve regressiecoëfficiënten

- **B** geeft de verandering in de logit (=log odds) van de afhankelijke variabele aan bij één eenheid verandering van X. Het model is lineair in de logits.
- **Exp(B)** is de multiplicatieve verandering in de odds met een eenheid verandering van X ten opzichte van odds baseline (=multiplicatieve intercept).
  - Exp(B) < 1: afname van odds
  - Exp(B) > 1: toename van odds
- Bij categorische variabelen kunnen we **exp(B)** interpreteren als een **odds-ratio [OR]** = verhouding tussen twee odds.

Multiplicatieve coefficienten en de odds-ratio OR

- Odds = exp (B0 + B1*X1)
- Odds = exp(B0)*exp(B1*X1)
- Als X= 0: odds = exp(B0)*exp(0) = exp(B0)
- Als X=1: odds = exp(B0)*exp(B1)
- **Odd Ratio OR**: exp(B0)*exp(B1) / exp(B0) = exp(B1)

Geen gestandaardiseerde B’s

- Anders dan bij OLS heeft logistic geen gestandaardiseerde coefficienten.
- B’s zijn daarom alleen met elkaar vergelijkbaar als hun eenheden vergelijkbaar zijn.
- Wil je toch gestandaardiseerde coefficienten hebben, dan zul je eerst zelf de X-en moeten standaardiseren (=voorzien van vergelijkbare meeteenheid).

Inferentiele statistiek

- Logistic geeft niet de bij OLS gebruikelijke T-toets: t = B/SE. Deze kun je wel zelf berekenen.
- Wald statistic is t^2. Vergelijk met Chi2 of F-tabel met 1, veel vrijheidsgraden. Kritieke waarde: 3.84.
- SE's behoren horen bij logits. Betrouwbaarheids-intervalllen rondom logits zijn symmetrisch, rondom multiplicatieve coefficienten zijn ze asymmetrisch.

Logistische regressie met nominale onafhankelijke variabelen

- Bij logistic behoef je niet zelf dummy-variabelen aan te maken by categorische X (het mag wel).
- **`../cat=X1 /contrast(X1)=indicator(1)`** geeft aan dat X1 categorisch is en 1 de referentie-categorie is.
- De output kan behoorlijk verwarrend zijn. Let goed op de “Categorical variable codings”.
- De Wald statistic is nu een test op gezamenlijke bijdrage van de dummy-variabelen.

Tabel ‘Categorical variable codings’

Categorical Variables Codings

			Parameter coding				
		Frequency	(1)	(2)	(3)	(4)	(5)
agecat	19	22	.000	.000	.000	.000	.000
	22	53	1.000	.000	.000	.000	.000
	30	276	.000	1.000	.000	.000	.000
	40	453	.000	.000	1.000	.000	.000
	50	379	.000	.000	.000	1.000	.000
	60	363	.000	.000	.000	.000	1.000

Homework

- Read O'Connell 1-27.
- Practice the use of binary logistic in outcome 'University education' with AGE and FEMALE in ESS, using LOGIST, NOMREG and PLUM. I will send around further specification.